"NON-LINEAR FORCED VIBRATION STUDY OF AXIALLY FUNCTIONALLY GRADED NON-UNIFORM BEAMS BY USING BROYDEN METHOD"

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF

Master of Technology

In

Machine Design and Analysis

By

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Department of Mechanical Engineering

National Institute of Technology
Rourkela Odisha
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Under Supervision of

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NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA

CERTIFICATE

*This is to certify that the thesis entitled, "**Non-Linear Forced Vibration Study Of Axially Functionally Graded Non-Uniform Beams By Using Broyden Method**" by **Virendra Mathpati** in partial fulfillment of the requirements for the award of Master of Technology Degree in Mechanical Engineering with specialization in "Machine Design & Analysis" at the National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.*

To the best of my knowledge the matter embodied in the thesis has not been submitted to any other University/ Institute for the award of any Degree or Diploma.

Date**:**

Prof. Anirban Mitra

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LIST OF SYMBOLS USED:

A	Cross-sectional area of Beam
{c}	Vector of unknown coefficients
[K]	Stiffness matrix
L	Length of the bar
Γ	Lagrangian
[M]	Mass matrix
nw	Number of functions for displacement field w
nu	Number of functions for displacement field u
T	Total kinetic energy
u	Displacement in axial direction.
w	Displacement in transverse direction.
ξ	Normalized coordinate in x direction.
U	Total strain energy
V	Work potential
{f}	Load vector.
ϕi	Displacement field due to bending
αi	Displacement field due to stretching

ABSTRACT:

In the forced vibration case due to external loading deflection tends to higher value so beam element undergoes the non-linearity behavior. Basically non linearity is categorized into two parts material and geometric non-linearity. Due to large amplitude we have to take care of both in-plane and out-plane displacement fields. With the help of variation form of energy principle all displacement fields calculated. These displacement fields are the combination of admissible orthogonal function. Admissible functions satisfies the both flexural and Membrane boundary conditions. By considering non-linear strain displacement relationship geometric nonlinearity is introduced in this thesis. The resulting nonlinear set of governing equations is solved through a numerical procedure involving direct substitution method using relaxation parameter. So calculation of natural frequency under both free and forced vibration decides the working condition of beam. Now a day in structural field tremendous growth is taking place for which we required a material which attains a good property and it became possible by functionally graded materials. Functionally graded material is non-homogenous and anisotropic material whose both structural and material property varies along the element. In FGM effect of residual stress and stress concentration is minimum in between two dissimilar materials which increase the strength and toughness of that structural element. So in this paper we are dealing with forced vibration analysis of FGM. For the static analysis we are using minimum potential energy principle and for the dynamic analysis we are using Hamilton's principle. Further non-linearity of beam can be calculated by using Broyden method. Through this research we obtained some results which are validated with some previous papers and then we submitted our further research results.

Key words- Axially functionally graded beam, forced vibration, frequency response, and Broyden method.

CHAPTER 1.

1.1 INTRODUCTION:

Vibration of structural element or any system is a great important factor in design concept. Due to vibration many times system may damage totally or partially. We have to find out all the causes of vibration system, So that we can avoid resonance of system. Resonance can cause unexpected damage of system. Resonance can occur when natural frequency of system is equal to excitation frequency of system. By using various methods we are calculating the frequencies of system, so that we can avoid damage of system.

Beams are most important structural element which can be widely used in various applications. In various complex structures we are using beam elements so it is one of the interesting areas of research. In the working area mostly we are using non uniform beams which are adopted for economical considerations. In the non-uniform beams due to aesthetic and erection considerations tapered beams are mostly used. Supporting conditions of beam are also one of the important considerations. Supports should be idealized for simple analysis.

Generally structural element can be defined as an element which can carry load in various directions. These elements can be in various shapes and of different materials. According to its load carrying capacity it can be subdivided into three parts- one dimensional element (bar),Two dimensional element (beam) and Three dimensional element (plate).Bar is a one dimensional element which has longitudinal or axial dimension much larger than the other two dimensions. Beam is an element which has tendency to carry load in the axial direction as well as in transverse direction, where as plate can carry load in all three directions.

In our day to day life we mostly interact with failure of beams due to various problems, so in this thesis we are concentrating on beams. Beam can be made in different shapes with use of different materials. As per the shape of beam it can be- uniform beam, taper beam. In taper beam again it is categorized as- linear taper beam, parabolic taper beam and exponential taper beam. By considering the various parameters of working area they are made by- homogenous material, composite material or functionally graded material. Normally any pure material can't give all satisfactory results so we needed some combination of different materials. But composite materials have some disadvantage of delaminating layers of it at very high temperature. So by considering creativity of nature new material functionally graded material has been founded for different applications. In the FGM all sharp interface of composite materials can be replaced by gradient interface which produces smooth interface. Also FGM can withstand at very high temperature. In the FGM material can be mixed in grades so that we can achieve drastic variation in the properties of material from one point to other point. Within FGM different microstructure phases have different functions so overall FGM obtain the multi structural status from their property gradation. According to that variation they are subdivided into two categories- axially based FGM and transverse based FGM. In the axially based FGM all the material properties vary in different proportions along the longitudinal axis. Whereas in the transverse based FGM all the material properties varies perpendicular to the longitudinal axis.

1.2 INTRODUCTION TO VIBRATION:

Vibration can be defined as to and fro motion of body about its equilibrium position. So according to its vibration initiation they can be categories into free vibration and forced vibration.

1.2.1 FREE VIBRATION:

When no any energy input is added to the system then that system will vibrate naturally known as free vibration. With the help of some input energy vibration of system started but as time goes it dies out by dissipating energy into surrounding. In this case, when any system is disturbed from their original position then there is some natural force which tries to keep that system in rest position.

1.2.2 FORCED VIBRATION:

When any disturbance is applied to the system and vibration occurs in that system with time varying disturbance then such vibrations are known as forced vibration. Input disturbance can be load, displacement or velocity. Also applied disturbance may be either periodic, steady state, transient or random input. A harmonic disturbance or non harmonic disturbances are known as periodic disturbance. Vibrations produced in transportation, vibrations due to imbalance of washing machine are some sort of applications. In the case of linear system frequency response of steady state condition due to periodic input is equivalent to frequency of applied motion or force. These response magnitudes are dependent on actual mechanical systems.

When any load is applied on the body then that body gets displaced to some position. So according to its deformation direction they can be considered as lateral deformation and transverse deformation. When the occurred displacement is in the direction of longitudinal axis then it is known as axial displacement. When that displacement is perpendicular to longitudinal direction then it is known as transverse displacement. So during large amplitude of vibrations we have t take care of both these amplitudes.

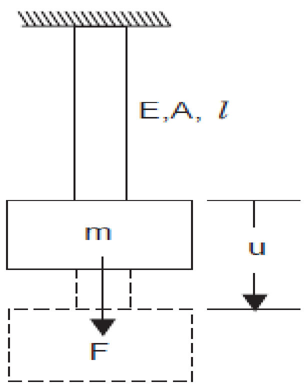


Figure 1.1 . (Linear Displacement of Beam)

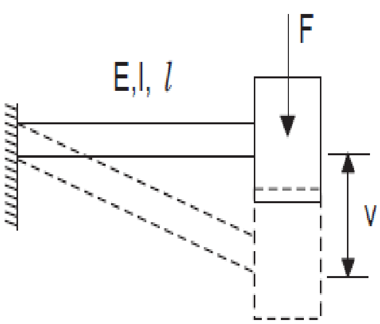


Figure1.2.(Transverse Displacement of Beam)

Also vibration motion can be considered as translation motion and rotational motion. In the translational motion body undergoes for stretching so that shape of body will remains constant but its volume get change. Whereas in the rotational motion body undergoes twisting so that shape change takes place. In this thesis we are considering only translational motion of body. No any rotational motion is considered for analysis of beams.

In generally all system exist as non-linear in the universe. For the sake of simple and easy way we are considering linear problems. When the displacement of beam became so high then it undergoes for the geometric non-linearity. In this case steady state response will not be proportional to excitation amplitude. Upto certain extend in the non-linear case frequency response function can be used to relate response of structural system to the excitation amplitude. In the nonlinearity harmonic excitations of the system causes periodic with non-harmonic or quasi periodic responses. Also superposition principle doesn’t hold for non-linearity. Sometimes not only the system response is non-linear function of excitation amplitude but also steady state solutions can be for same frequency and excitation amplitude.

1.3 INTRODUCTION TO NON-LINEARITY:

Linear vibration analysis can be carried out by considering small amplitude vibrations of long objects. Those objects can be long bridges, helicopter blades, airplane wings. Also rocking motions formed in ships, whirling motions produced by the flexible shafts. But if we considered an analysis of bridges and foundations, wings/blades and air, between ships and waves, between shafts and bearings are all nonlinear.

Some typical behavior of nonlinearity can be explained as-

- Various steady state responses, some formed stable and some formed unstable, during same inputs response.
- By varying forcing parameter we can obtain significant change in the response, also it contains some sort of discontinuity which introducing the jump phenomena.
- Response obtained at frequencies other than forcing frequencies, Internal resonances, it involves different parts from the system vibrating which are vibrating at different frequencies.
- Self sustained oscillations in the absence of explicit external periodic forcing.
- Complex, irregular motions those are extremely sensitive to initial conditions.

Basically non linearity is categorized into two parts material and geometric non-linearity. In the geometric non-linearity deflection of structure is very large compared with original dimensions (Non Linear Strain-Displacement relation), where as material non-linearity is associated with inelastic behavior (Non Linear Stress-Strain relation) of element. Geometric nonlinearity is caused by large amplitude transverse displacement and it induces a stretching effect, which provides additional stiffening to the structure. However, due to inclusion of the stretching effect the dynamic response becomes a nonlinear function of amplitude of excitation.

1.4 HISTORY OF FGM:

Functionally graded material structures are somehow relates with the nature. Plants, bones and teeth of human body to bio-tissues of animals are related with nature. Dental crown can be one of the examples of FGM application. Its structure requires outside portion very high wear resistance, whereas inner portion should be soft enough to sustain fatigue and brittle fracture. Also if we considered aesthetic look it requires some color nuances and outer area as translucent.

Functionally graded material first found out by Japanese researchers in 1984-85. When they were working for spaceplane project then they required some advanced material which can sustain temperature more than 1000°C. Normally spaceplane exposed to very high temperature about (1700°C) environment. So to sustain such temperature some non-uniform material required. For that purpose they fabricated a material whose composition will change gradually, so we will get advantageous of both mechanical properties and thermal resistance.

In the applications of thermal –barrier structures, wear resistant and corrosion resistant coatings functionally graded material suited well. Also by joining two dissimilar metals we can have possibility of delaminating but FGM don’t have such possibility so it works well under such conditions.

FGM has occupied a very large application area. They are using in plates and shells, reactor vessels. Also some machine parts which undergoes for failure like buckling, excessive stress due to various loading at same instant. If we applied FGM coating over these structures we can reduce the percentage of such failures.

1.5 APPLICATION-

A. BEAMS-

Beam is one of the important structural parts which have wide application area. They can be used in marine applications, some moving parts and in structures. Also they are used for smaller frame structures which are used in mechanical system, automobile trucks and in some machines. In this also as weight goes on increasing other factors affect on the structure so mostly in complicated structures we are using thin beams. Some sections made like rectangular tubes, rounded type or square type. These sections may be of open type or closed type. Open sections can be known by I-type, L-type or T-type beams. Thin sections exhibit high bending stiffness for a unit area than rod so they are mostly preferred than rod or bar. So according to that high stiffness can be achieved by low weight structure.

B. FGM-

Functionally graded materials are mostly made by combining ceramic and metals so they give good thermal and strength properties. These materials are more popular in various applications cause of their existing properties which are more convenient for engineering applications. Few applications are listed below-

- ❖ Aerospace- In aerospace to withstand very high thermal gradient FGM can be used. To make rocket engine component and space plane body various functionally graded materials used.
- ❖ Medicine – living tissues like bones and teeth are characterized as FGM, so all death tissues can be replaced by FGM.

Also cutting tool material inserts coating, automobile engine coating, nuclear reactor components and turbine blades are made by functionally graded materials.

CHAPTER 2.

2.1 LITERATURE REVIEW:

The problem of bending of beam has been one of the subjects of investigation. To calculate that deformations and natural frequencies under both free and forced vibration tremendous research takes place on beam. In the following paragraphs a brief review of some of the relevant research work has been provided.

Ahmad Shaba et al. [1] considered the Euler- Bernoulli tapered beam with axially FGM. By using governing equations of motion they studied the free vibration analysis and stability of beam. They found out that Differential transform element method can converge the results more satisfactorily than the differential transform method. Also by using differential quadrature element method we can solve the governing equations. With the help of various examples they verified the competency of DQEL and DTEM in the analysis of free longitudinal and transverse frequencies and also critical buckling load of non uniform beam.

Bayat et al. [2] analyzed the non-linear free vibrations of beams. Free vibration analysis of taper beam is carried out by considering governing equations. To get the natural frequency of system and all displacement field of the system they applied new ancient Chinese method called as Homotopy Perturbation Method and -Min Approach. By comparing obtained results with exact method effectiveness and convenience of these methods are illustrated. They also concluded that these methods are simple and easy. In single iteration we can obtain high accuracy.

During transverse motion when speed of the system approaches to critical speed then non-linearity of system has been one of the major tasks. By using Lindsted-Poincar´e method Wickert [3] calculated the natural frequencies of travelling beam under non-linear condition. When we will decouple the governing equations from coupled transverse and longitudinal motion then non-linearity can be derived. In this case we are assuming that tension of the beam is not going to change in the longitudinal direction. The non-linearity obtained in this case will be of integro-partial-differential. Without assuming quasi-static stretch condition partial-differential equations can be derived for transverse displacement.

C.N.Chen [4] assumed the Hamilton's principle. He also derived the dynamic equilibrium equations of non-uniform beams with considering various different cross-sections. This system is defined by selecting arbitrary co-ordinate system. The analysis is carried out by coupling natural boundary conditions and dynamic equilibrium equations. All the dynamic equilibrium equations are obtained by using Hamilton's principle. Further formulation is carried out by using variational approach.

Hoseini [5] examined the homotopy analysis method. By considering this method he found out the natural frequencies and displacement of non-uniform beam undergone for non-linearity. When they compared the results with other techniques then they observed that this method gives accurate solution and relative error is also less than 0.5%. So they proved that HAM is the powerful technique which gives convenient and effective mathematical tool to obtain all non-linear differential equations. Beside all the advantages of the HAM, there are no rigorous theories to direct us to choose the initial approximations, auxiliary linear operators, auxiliary functions, and auxiliary parameter.

Das et al. [6] studied the dynamic behavior of non uniform rotating beam under the free vibration case in both elastic and plastic stage. They used in this case substitution technique, in which by using the displacement of static analysis further displacement of dynamic analysis is carried out. The governing equations used for analysis are calculated by variational principle.

Yoo and shin [7] presented the vibration study of rotating type cantilever beams by adding the effect of gyroscopic couple.

Park and Kim [8] examined the dynamic analysis of non-uniform rotating beam having tip mass and undergone for non-linearity.in the analysis they also considered centrifugal force, Corioli's component force and acceleration of system.

Mesut Simsek [9] considered the functionally graded beam undergoing for non-linear dynamic analysis. Also in this analysis he considered Timoshenko beam theory and supporting conditions are pinned–pinned. For further analysis he considered von Kerman's non-linear strain displacement relationship. System equations needed for analysis are generated with the help of Lagrange's equations.

Pradhan et al. [10] found out the thermo-mechanical vibration analysis of functionally graded beams and functionally graded sandwiched beams resting on variable Winkler and two parameter elastic foundations by using Euler Bernoulli's beam theory and modified differential quadrature method.

Kitipornchai et al.[11] studied the non linear free vibration of functionally graded Timoshenko beams containing an open edge crack based on von Kerman geometric non linearity.

Ansari et al.[12] examined the forced vibration behavior of nano composite beams made by single walled carbon nanotubes based on Timoshenko beam theory is carried out by von Kerman's non linearity.

Ke et al.[13] presented analytical solution for the linear free vibration and buckling characteristics of functionally graded beam with open crack by using Timoshenko beam theory.

The lot of investigation has been devoted to the linear dynamic analysis of various structures. In these cases the stiffness matrix is complex one which depends on the vibration frequency as non-linearity. Linear vibration analysis leads to the non-linear vibration analysis which tends to complex Eigen problem. The solution of such type of problem leads to complex modes which are little beat different from undamped modes and second thing is that complex Eigen value whose imaginary and real parts are somehow related with loss factors and viscous Eigen frequency. According to engineering point most relevant quantity is loss factor which is associated with any mode.

Kojima[14] deals with the forced vibrations of a beam with a non-linear dynamic vibration absorber are investigated. The beam is simply supported at both ends and subjected to a sinusoidal motion of constant amplitude; the damping in the beam is neglected. The restoring force of the non-linear dynamic absorber is represented by a hardening spring having both linear and cubic non-linear behavior. In the analysis the third order super harmonic and the one-third order sub harmonic as well as the fundamental are assumed to be present. By applying the harmonic balance method, non-linear simultaneous equations are obtained, and these are solved by using the Newton-Raphson technique. Furthermore, it is demonstrated that the optimal tuning and damping parameters for a non-linear magnetic absorber with a hardening spring can be easily found by using the simplex method.

Daya et al.[15] established the simplest consistent theory for the non-linear vibration analysis of a viscoelastic sandwich beam. In the elastic face layer axial stretching is occurred which causes non-linearity. In the viscoelastic layer due to shear deformation damping arises. With the help of simple constituent theory sandwich model is treated. In this analysis harmonic balance method is coupled with one mode Galerkin approximation. The obtained amplitude equation is similar to classical bifurcation equation.

Gunda and Ganguli [16] considered a new rotating beam. In this beam interpolating shape functions are functions of element position and rotational speed. This also accounts the centrifugal stiffening effect.

Lee et al. [17] studied the in-plane free vibration behavior of a rotating curved beam with an elastically restrained root.

Banerjee [18] developed the dynamic stiffness matrix of a centrifugally stiffened Timoshenko beam and applied Frobenius method of series solution to solve for natural frequency of rotating Timoshenko beam.

S.M.Ibrahim et al. [19] Considered three noded beam elements for experimentation. The beam element is based on higher order shear deformation theory which satisfying interlayer’s continuity displacement and transverse shear stress. Also top and bottom boundary condition is used for finite element formulation.

S.stoykov [20] considered the isotropic beam element with arbitrary cross section .for the bending analysis Timoshenko beam theory is applied where as for torsion saint venant's principle is applied. Also in this warping function of beam can be calculated by solving Laplace equation with Neumann boundary condition using boundary element method.

Ribeiro [21] demonstrated the feasibility of finite element, shooting and Newton methods in the determination of non linear periodic motion for thick or thin beam. By doing this research he demonstrated that suggested methods highly adequate to analyze the periodic forced non-linear dynamics of beam structure.

Formica et al. [22] performed a study on the vibration characteristics of CNTRC plates and discovered that the improvement reaches a maximum when CNT's are uniformly aligned along the loading direction.

T. Pirbodaghi et al. [23] introduced a new homotopy analysis method. With the help of this method he carried out the non-linear vibration of Euler-Bernoulli's beam which is subjected to axial loads. Also by using this method he proved that HAM high accurate solutions than other methods so that we can use it for wide applications.

So till now research carried out for transverse functionally graded material. Also various finite element methods used to analyze. So in this paper we are studying the non linear forced vibration analysis of axially functionally graded non uniform beams.

In this thesis we are considering large displacement of beam undergoes for a non linear forced vibration case. Beam is subjected for harmonic excitation. When we are considering the dynamic system then peak value of excitation amplitude is considered in force equilibrium condition. By considering this assumption overall dynamic problem can be converted to static analysis. With the help of appropriate governing equations energy methods applied for the system. Also in this method we are considering geometric non-linearity with the help of non-linear strain displacement relations. In this formulation part all the unknown displacement fields are nearly approximated with the help of admissible orthogonal functions. All formulation is carried out by considering displacement field only. The overall formulation is based on the whole domain analysis. For the solution purpose in this work multi dimensional quasi-Newton method which is also known as Broyden method is applied.

In the analysis of vibration we have to calculate some displacements at all field for which we needed some methods which are stated below. In this analysis static deflection can be carried out by using minimum potential energy methods. Dynamic analysis of beam is carried out by using Hamilton's principle. To get the better non-linearity in the work we are using Broyden method for collecting higher vibration amplitude values in some region and some lower vibration amplitude value in some region. Also to carry out further analysis some energy methods are used which are explained below.

2.2 Motivation:

Functionally graded material has been one of the important materials in industry. Till now research shows that most of the analysis is carried out on the basis of transverse FGM. So in this present work by considering axially functionally graded material the analysis is done. Also by considering geometric non-linearity case the analysis has been carried out by implementing broyden method. In this thesis we considered various geometric conditions of beam which are mostly using now a day for better considerations. Also various types of variation in material elasticity and density are taken. By considering four types of boundary condition and various material variation and geometry variation frequency response curves obtained in the analysis.

2.3 Goal of Present Work:

The specific purpose of the present work of thesis have been laid down as

- ✓ To adapt energy principle in formulation of the problem.
- ✓ Study the forced vibration behavior of non-uniform beam by using axially functionally graded beam with different boundary condition.
- ✓ Validated our result by Broyden method with previous obtain result in research by finite element method.
- ✓ To compare the frequency response curve of different functionally axially graded material for specific boundary condition and specific variation of thickness.

CHAPTER 3.

3.1 ENERGY METHODS:

To find out the deformations of any structure we can use energy methods. It's simple to use and advantageous over other methods. This is simply based on linear elastic behavior of structure and first law of thermodynamics. It states that the energy stored in the structure under certain load is equal to work done by all external forces.

Energy method gives the relation between stress and strain. Also by this method deformation and displacement can be presented in the form of energy or work done. Work done on the structure can be due to internal or external forces. So it is utilized by two fundamental scalar quantities work and energy.

The advantage of energy method over other is formulation of this method is done by using generalized co-ordinates. Also we can use it for approximate solutions complex systems, by giving difficult task of solving the set of governing partial differential equations. Following are some energy methods-

3.2 MINIMUM POTENTIAL ENERGY PRINCIPLE:

This method is generally used for static analysis of any structure. All the deformations and stress analysis of any type of structure can be analyzed by minimum potential energy principle.

It can be stated as:

"For conservative structural systems, of all the kinematially admissible deformations, those corresponding to equilibrium state extremize (i.e. minimize or maximize) the total potential energy. If the extremum condition is minimum then the equilibrium state is stable."

Deformation is unknown principle and strain is depend on deformation, where as stress is depend on strain. Deformation refers to incremental change to deformed state from original undeformed state.

The total potential energy (π) of a body is defined as the sum of total strain energy (U) and the work potential (V), i.e.

$$U+V=\pi$$

According to minimum potential energy principle, the equilibrium condition of the system is obtained by letting $\delta(\pi)=0$ Implementation of minimum potential energy principle in structural mechanics problem was first addressed by Lord Rayleigh and later it was extended by Ritz, to its final form known as Rayleigh-Ritz method.

3.3 RAYLEIGH RITZ METHOD:

In this method displacement fields are assumed as admissible. Maximum kinetic energy is equated with the maximum total potential energy which results in upper bound estimation of fundamental natural frequency. The function is stated as admissible when it satisfies the boundary conditions, compatibility equations and it should be continuously differentiable.

According to Rayleigh-Ritz method, the components of the approximate displacement field are taken to be of the form given below-

$u = \Phi_0^1(x,y,z) + \sum_{i=1}^{l} c_i * \Phi_i\,(x,y,z)$ l= no. of functions for u

$u = \Phi_0^2(x,y,z) + \sum_{i=1+l}^{m} c_i * \Phi_i\,(x,y,z)$m=no. of functions for v

$u = \Phi_0^3(x,y,z) + \sum_{i=1+m}^{n} c_i * \Phi_i\,(x,y,z)$ n =no. of functions for w

Where, u, v & w are the components of displacement in x, y & z direction.

$\Phi_0^\alpha\ (\alpha = 1,2,3)$= Coordinate functions satisfying the specified (in their actual form) essential boundary conditions associated with u, v and w respectively,

$\Phi_i^\alpha\ (\alpha = 1,2,3)$= Linearly independent and complete sets of continuous coordinate functions satisfying the homogeneous form of specified boundary conditions associated with u, v and w respectively,

c_i= Undetermined coefficient.

$$u = \sum_{i=1}^{l} c_i * (\Phi_0^1 + \Phi_i) = \sum_{i=1}^{l} c_i * \Phi_i$$

$$u = \sum_{i=1+l}^{m} c_i * (\Phi_0^2 + \Phi_i) = \sum_{i=1+l}^{m} c_i * \Phi_i$$

$$u = \sum_{i=1+m}^{n} c_i * (\Phi_0^3 + \Phi_i) = \sum_{i=1+m}^{n} c_i * \Phi_i$$

So, Φ_i^{α}(α=1, 2, 3) satisfy the boundary condition.

Most important consideration in this method is that all displacement fields should be kinematically admissible. In other case we can supply some initial displacement by considering some assumed form. The condition dealing with initial displacement is that it should not introduce any type of geometric discontinuity. Also it should not violate any type of geometric constraints. In the beam problem when we are considering rigid support then it implies that displacement should be zero. While considering built-in supports in problem slope should be zero at that point. Except those boundary points slope and displacements must be continuous throughout the length of that beam.

3.4 GALERKIN METHOD:

Galerkin method is a weighted residual method in which the approximate solution is sought using weighted integral statement of the equation. Here, as mentioned in the previous paragraph, the assumed solution is also in the form of a finite linear combination of undetermined parameters or coefficients with appropriately chosen coordinate functions. A special characteristic of Galerkin method is that the approximate functions and the weighted functions are the same. Since the solution of a continuum problem in general cannot be represented by a finite set of functions, an error (called the residual, ε) is induced when the assumed solution is substituted in the governing equation. The residual is minimized by using the coordinate functions as the weight functions. In this method also substitution of the assumed solution into the governing equations leads to a set of algebraic equations of the form $[K]\ \{d\} = \{f\}$. If the set of functions ϕ_i is selected from an orthogonal set, the solution becomes simpler. Throughout the present thesis work this has been accomplished by the use of Gram Schmidt orthogonalization scheme.

3.5 Gram Schmidt Orthogonalisation Scheme:

The start functions for the assumed solution, i.e., displacement fields in case of the current work, are selected by satisfying the boundary conditions of the problem. In order to set up an orthogonal set of functions, the higher order functions are generated from the chosen start functions through a numerical implementation of Gram Schmidt orthogonalization scheme. For the convenience of the numerical scheme, all the functions are defined numerically at some suitably selected Gauss points. The details of the procedure of generation of the set of functions in one dimension are furnished here (Saha et al., 2004). Starting with a polynomial $\phi_1(x)$, an orthogonal set of polynomials in the interval $a \le x \le b$ can be generated by the following scheme.

$$\phi_2(x) = (x - B_1)\,\phi_1(x)$$

$$\phi_k(x) = (x - B_k)\,\phi_{k-1}(x) - C_k\phi_{k-2}(x)\text{, where,}$$

$$B_k = \int_a^b x w(x)\,\phi_{k-1}^2(x)\,dx \Big/ \int_a^b w(x)\,\phi_{k-1}^2(x)\,dx$$

$$C_k = \int_a^b x w(x)\,\phi_{k-1}(x)\,\phi_{k-2}(x)\,dx \Big/ \int_a^b w(x)\,\phi_{k-2}^2(x)\,dx$$

with $w(x)$ being the weight function. The set of function $\phi_k(x)$ satisfies the orthogonality condition given by,

$$\int_a^b w(x)\,\phi_k(x)\,\phi_l(x) = \begin{cases} 0 & if\ k \ne l \\ 1 & if\ k = l \end{cases}$$

For the present thesis work, the weight function is chosen as unity and the interval is from 0 to 1. For two dimensional problems, two dimensional implementation of this scheme has been developed and used successfully by Das et al. (2009). In the present work the two dimensional Gram Schmidt procedure has been utilized for generation of the higher order functions. While approximating the displacement fields for the two dimensional static problems it is assumed that the coordinate functions are separable in two coordinate directions, thus making the formulation simpler. For the dynamic problems, the coordinate functions are assumed to be separable in space and time.

3.6 Solution of Eigen value Problem:

The free vibration analysis of stiffened plates has been formulated as a standard Eigen value problem of the form, $-\omega^2[M]\{d\}+[K]\{d\}=0$ and its solution is acquired numerically through IMSL routines. The linear buckling analysis is also performed as an Eigen value analysis. Out of many available routines for Eigen value analysis, the EIGRF routine, which computes Eigen values and the associated eigenvectors of a real general matrix in full storage mode, is utilized to obtain the solutions. EIGRF routine uses a well known Eigen value algorithm in numerical linear algebra, called QR algorithm, to calculate the Eigen values and eigenvectors. The QR algorithm was developed in the late 1950s independently by John Francis (1961, 1962) and by Vera N. Kublanovskaya (1963). The basic idea of this algorithm is to perform a QR decomposition, which writes a matrix as a product of an orthogonal matrix and an upper triangular matrix, $[A] = [Q]\ [R]$. Here $[Q]$ is the orthogonal matrix and $[R]$ denotes the upper triangular matrix, which is also known as the right triangular matrix. There are several methods for actually computing the QR decomposition, such as by means of the Gram–Schmidt process, Householder transformations, or Givens rotations. In QR algorithm after the decomposition, reverse order multiplication along with simultaneous iterations is performed to find out the Eigen values.

3.7 Broyden’s Method:

Broyden's method is a quasi-Newton method for the numerical solution of nonlinear equations in more than one variable. It was originally developed by C. G. Broyden in 1965 and is often described as a multidimensional secant method. This method utilizes the Jacobian matrix and instead of computing the whole Jacobian matrix at every step of the iteration, performs only an update by making the least change to the previous Jacobian. The difficulty of computing the Jacobian matrix is very well known. If the functions are sufficiently simple for their partial derivatives to be obtained analytically, the amount of computations required may well be excessive. In the majority of practical problems, however, the functions are far too complicated and an approximation to the Jacobian matrix must be obtained numerically. In the method described by Broyden (1965) the partial derivatives are not estimated or evaluated directly, but corrections to the approximate Jacobian matrix $[B]$ are computed from values of the function. A correction of this type is carried out at subsequent iterations following the expression given below and is much simpler to perform than the evaluation of the complete Jacobian. The algorithm and subsequent codes for Broyden's method used in the present thesis work is given by Press et al. (2005).

$$[B]^{i+1} = [B]^{i} + \frac{([\delta F]^{i} - [B]^{i}.\{\delta x\}^{i}).(\{\delta x\}^{i})^{T}}{\{\delta x\}^{i}.\{\delta x\}^{i}}, \text{ where, } \begin{cases} [\delta F]^{i} = [F]^{i+1} - [F]^{i} \\ \{\delta x\}^{i} = \{x\}^{i+1} - \{x\}^{i} \end{cases}$$

3.8 NUMERICAL IMPLEMENTATION OF SOLUTION SCHEME:

Variational methods convert governing differential equation into set of algebraic equations. According to nature of problem they can be exist as linear or nonlinear problem. When any load vector or any stiffness matrix is dependent on displacement field then it can be treated as nonlinear problem.

The entire static problem can be solved by $[\mathrm{K}]\{\mathrm{u}\} = \{\mathrm{f}\}$

Whereas $[\mathrm{K}]$ and$\{\mathrm{u}\}$are independent on displacement field.

There are various numerical solution methods of multidimensional problem to get both linear and nonlinear set of equations.

3.9 HAMILTON'S PRINCIPLE:

The principle of virtual work and minimum total potential energy are limited to problems of static equilibrium of deformable bodies and are unable to deal with problems of dynamics on their own. However it is well known that D'Alembert's principle states that a system can be considered to be in equilibrium if inertia forces are taken into account, thereby reducing a dynamic problem into a problem of statics. By the use of D'Alembert's principle along with the principle of virtual work governing equations of the dynamic problem can be derived in a manner similar to the static case, wherein the virtual work done by the inertial forces are now included (Washizu, 1982). Most widely used principle utilizing the above mentioned scheme is the Hamilton's principle, which is basically a generalization of virtual work principle to dynamics. This principle considers the motion of the entire mechanical system between two finite time instants and is therefore an integral principle. Motion of the system is described through a scalar potential, which may depend on the coordinates, velocities and time. Thus the dynamic problem is reduced to an investigation of a scalar definite integral. One advantage of this principle is that it is invariant with respect to the coordinate system used.

The statement of the Hamilton's principle is: "of all the paths of admissible configuration that the body in motion can take as it goes from configuration 1 at time τ_1 to configuration 2 at time τ_2, the actual configuration is the path that extremizes time integral of the Lagrangian (ζ) during the interval" (Shames and Dym, 1985). Mathematically Hamilton's principle is expressed as, $\delta\left(\int_{\tau_1}^{\tau_2} \zeta \, d\tau\right) = 0$, where, $\zeta\,(= T - \pi)$ is known as the Lagrangian and T is the total kinetic energy of the system. It is clear that the principle characterizes the system under consideration by two energy functionals, kinetic energy (T) and total potential energy ($\pi = U + V$). In the present work dynamic problem of stiffened plates has been formulated through Hamilton's principle.

3.10 STARTING FUNCTION GENERATION:

The start function for clamped-clamped beam is calculated as:

$W(x) = C_0 + C_1x + C_2x^2 + C_3x^3 + C_4x^4$ (4.1)

Boundary condition for clamped-clamped can be mathematically expressed as,

At $x = 0$ and L, $w = 0$ and $\frac{dw}{dx} = 0$

So, putting the values corresponding to $x = 0$: $c_1 = 0$ and $c_0 = 0$;

By putting the above value in general equation (4.1)

At $x = L$, $w(x) = 0$;

$C_2 = -(C_3L + C_4L^2)$ (4.2)

At $x = L$,

$\frac{dw}{dx} = 0$

$2C_2 + 3C_3L + 4C_4L^2 = 0$;

$C_2 = -(3C_3 L + 4 C_4 L^2)$ (4.3)

From (4.2) & (4.3)

$C_2 = C_4 L^2$

$C_3 = -2C_4 L$

$w = C_4 L^2 x^2 - 2 C_4 Lx^3 + C_4 x^4$

Inserting the normalized coordinate, ξ, in place of x and also replacing L by 1;

$w = C_4 \xi^2 (1-2\xi+\xi^2)$

Considering c4 as constant and equal to unity final form of the start function corresponding to clamped-clamped boundary condition becomes, $w = \xi^2 (1-\xi)^2$.

Table 3.10 (Start Functions)

Boundary conditions	Start functions
Clamped –clamped	$(\xi(1-\xi))^2$
Simply supported	Sin (pi* ξ)
Clamped-free	ξ^2 *(6.0-4.0* ξ + ξ^2)
Clamped-supported	ξ^2 *(3.0-5.0* ξ +2.0 ξ^2)

After generating starting function we required is the energy equation which can be obtained by calculating strain energy of non-uniform beam in both in-plane and out-planes displacement fields. Also Hamilton's principle is applied on the beam in which we are substituting kinetic energy and potential energy of beam.

CHAPTER 4.

4.1 BEAM FORMULATION:

The static analysis of beam can be derived by minimum potential energy principle. In the minimum potential energy principle the stain energy and potential energy of the system calculated. So in this work we calculated the strain energy of beam. When the deformation of beam is so large then in plane and out of plane both takes place, so we have to consider both stretching and bending of the beam.

$$U=U_b+U_m \tag{4.1}$$

We know the strain energy of structure-

$$U_e = \frac{E}{2} * \int_0^l \varepsilon^2 * A * dx \tag{4.2}$$

In the above equation, strain energy is directly proportional to square of strain. So strain of the beam can be calculated by stretching and also by bending. Strain of the structure is depending on deformation. Strain of the beam can be represented as,

$$\varepsilon = \frac{du}{dx} + \frac{1}{2} * \left(\frac{dw}{dx}\right) - y * \frac{d^2w}{dx^2} \tag{4.3}$$

As we know the beam is two dimensional elements so deformation will be in both longitudinal and transverse direction. So as to get that deformation we have to substitute this strain value in the stain energy equation.

$$\varepsilon^2 = \left(\frac{du}{dx} + \frac{1}{2} * \left(\frac{dw}{dx}\right) - y * \frac{d^2w}{dx^2}\right)^2 \tag{4.4}$$

$$\varepsilon^2 = \left(\frac{du}{dx}\right)^2 + \frac{1}{4} * \left(\frac{dw}{dx}\right)^4 + y^2 * \left(\frac{d^2w}{dx^2}\right)^2 + \left(\frac{du}{dx}\right) * \left(\frac{dw}{dx}\right)^2 - 2 * y * \left(\frac{du}{dx}\right) * \left(\frac{d^2w}{dx^2}\right) - y * \left(\frac{d^2w}{dx^2}\right) * \left(\frac{dw}{dx}\right)^2 \tag{4.5}$$

As area is varying along longitudinal direction it is also a part of integration, so it can’t be treated as constant. According to shape profile of beam its area relates with coordinate which is going to be change.

$$U_e = \frac{1}{2}\int_0^1\left(\left(\frac{du}{dx}\right)^2 + \frac{1}{4}\left(\frac{dw}{dx}\right)^4 + y^2 * \left(\frac{d^2w}{dx^2}\right)^2 + \left(\frac{du}{dx}\right) * \left(\frac{dw}{dx}\right)^2 - 2y * \left(\frac{du}{dx}\right) * \left(\frac{d^2w}{dx^2}\right) - y * \left(\frac{d^2w}{dx^2}\right) * \left(\frac{dw}{dx}\right)^2\right) * A(x) * E(x) * dx\,, \tag{4.6}$$

So in the above equation we are multiplying each term with area separately, so that we get some important terms which are used for further analysis and some terms going to vanish which can be neglected in further analysis.

We know for any beam structure,

$$y^2 * A(x) = I(x) \tag{4.7}$$

Also,

$$y * A(x) = 0.0 \tag{4.8}$$

Therefore substituting the above equations in the strain energy equation, then we get-

$$U_e = \frac{E}{2} * \int_0^1\left(A(x) * \left(\frac{du}{dx}\right)^2 + \frac{A(x)}{4} * \left(\frac{dw}{dx}\right)^4 + I(x) * \left(\frac{d^2w}{dx^2}\right)^2 + A(x)\left(\frac{du}{dx}\right) * \left(\frac{dw}{dx}\right)^2\right) * dx \tag{4.9}$$

For further calculations to make simple we are normalizing the longitudinal axis with the length of beam in that direction.

Now

$$\frac{x}{l} = \xi$$
$$x = l * \xi$$
$$x = 0, \xi = 0$$
$$x = l, \xi = 1$$

So the strain energy equation can be changed to,

$$U_e = \int_0^1\left(A(\xi) * \left(\frac{du}{d\xi}\right)^2 * \frac{1}{l^2} + \frac{A(\xi)}{4} * \left(\frac{dw}{d\xi}\right)^4 * \frac{1}{l^4} + I(\xi) * \left(\frac{d^2w}{d\xi^2}\right)^2 * \frac{1}{l^4} + A(\xi)\left(\frac{du}{d\xi}\right) * \left(\frac{dw}{d\xi}\right)^2 * \frac{1}{l^3}\right) * E(\xi)d\xi \tag{4.10}$$

So by keeping respective values of displacement fields we will get final strain energy equation. From that equation we can obtain the stiffness matrix.

Dynamic analysis of beam can be done by using Hamilton's principle.

$$\delta\left(\int_{\tau 1}^{\tau 2} \mathrm{L}\, d\tau\right) = 0 \tag{4.11}$$

In this case L is a lagrangian function which can be calculated by-

$$L = \{T - (U+V)\} \tag{4.12}$$

Whereas T=total kinetic energy of the system.

U= Potential energy of the system.

V= external work done on the system.

So, total kinetic energy of the system can be calculated as,

$$T = \frac{1}{2} b \int_0^l \left\{ \rho(\xi) b(\xi) \left(\frac{\partial w}{\partial \tau} \right)^2 + \left(\frac{\partial u}{\partial \tau} \right)^2 \right\} d\xi \tag{4.13}$$

The work potential needed for the system can be expressed as below-

$$V = -Pw - \int_0^L pwdx \tag{4.14}$$

So by keeping all the values of kinetic energy, potential energy and work potential we will get stiffness matrix element.

The following properties of a rectangular beam cross section about the neutral axis have been

Considered:

$$b\int_{-t/2}^{t/2} dy = A(\xi) \qquad b\int_{-t/2}^{t/2} ydy = 0 \qquad \text{and} \qquad b\int_{-t/2}^{t/2} y^2 dy = I(\xi)$$

The unknown dynamic displacements $w(\xi, \tau)$ and $u(\xi, \tau)$ are assumed to separable in space and time the spatial part of the fields are approximated by finite linear combination of admissible orthogonal functions identical to those utilized for static analysis,

$$w_x(\xi) \cong \sum_{i=1}^{nw} d_i \phi_i(\xi) \sin(\omega\tau) \tag{4.15a}$$

$$u_x \cong \sum_{i=nw+1}^{nw+nu} d_i \alpha_{i-nw}(\xi) \sin(\omega\tau) \tag{4.15b}$$

Where, ω is the natural frequency of the system and d_i represents a new set of unknown coefficients that represents the eigenvectors in matrix form. Where ϕ_i and α_{i-nw} denote the sets of orthogonal admissible functions for w_x and u_x, respectively, nw and nu denote the number of functions for w_x and u_x, respectively,

Harmonic Loading is given by = P sin ($\omega\tau$) (4.16)

By keeping all the respected values in equation 4.11 we can do the dynamic analysis of beam.

$$[K]\{c\} - \omega^2 [M]\{c\} = \{f\} \tag{4.17}$$

4.2 STIFFNESS MATRIX ELEMENT:

By substituting the total energy equation in the approximate displacement field then we will get expression,

[K] {d} = {f} (4.21)

Whereas in this equation,

[K]= stiffness matrix,

{d} = vector of unknown coefficients,

{f}= load vector.

In this problem we are considering nu, nw as number of displacement in respectively axial and transverse direction. So whatever matrix or vector we are obtaining that will be of nw + nu.

In this problem as we stated the problem of in plane and out plane displacement so we will get two stiffness matrixes. On stiffness matrix belongs to stretching and one stiffness matrix belongs to bending. Final expression can be expressed as,

$$[K] = [K_b] + [K_m] \tag{4.22}$$

$$[K_{11}] = \frac{E}{l^3}\sum_{i=1}^{nw}\sum_{j=1}^{nw}\int_0^1 I(x) * \left(\frac{\partial^2\phi_i}{\partial\xi^2}\right) * \left(\frac{\partial^2\phi_j}{\partial\xi^2}\right) * d\xi + \frac{E}{2l^3}\sum_{i=1}^{nw}\sum_{j=1}^{nw}\int_0^1 A(x) * \left(\sum_{i=1}^{nw} d_i * \left(\frac{\partial\phi_i}{\partial\xi}\right)\right)^2 * \left(\frac{\partial\phi_i}{\partial\xi}\right) * \left(\frac{\partial\phi_j}{\partial\xi}\right) * d\xi + \frac{E}{l^2}\sum_{i=1}^{nw}\sum_{j=1}^{nw}\int_0^1 A(x) * \left(\sum_{j=1+nw}^{nw+nu} d_i * \frac{\partial\alpha_{i-nw}}{\partial\xi}\right) * \left(\frac{\partial\phi_i}{\partial\xi}\right) * \left(\frac{\partial\phi_j}{\partial\xi}\right) * d\xi \tag{4.23}$$

$$[K_{12}] = 0 \tag{4.24}$$

$$[K_{21}] = \frac{E}{2l^3}\sum_{i=1+nw}^{nw}\sum_{j=1+nw}^{nw+nu}\int_0^1 A(x) * \sum_{i=1}^{nw} d_i\left(\frac{\partial\phi_i}{\partial\xi}\right) * \left(\frac{\partial\phi_j}{\partial\xi}\right) * \left(\frac{\partial\alpha_{i-nw}}{\partial\xi}\right) * d\xi \tag{4.25}$$

$$[K_{22}] = \frac{E}{2l}\sum_{i=1+nw}^{nw+nu}\sum_{j=1+nw}^{nw+nu}\int_0^1 A(x) * \left(\frac{\partial\alpha_{i-nw}}{\partial\xi}\right) * \left(\frac{\partial\alpha_{j-nw}}{\partial\xi}\right) * d\xi \tag{4.26}$$

In the case of dynamic analysis and whenever we will change the material the stiffness matrix of the beam will exhibit a different value but a mass matrix will remain same throughout the analysis which is expressed below-

4.3 MASS MATRIX ELEMENT:

In whole analysis due to excitation stiffness value will be changed but mass matrix element will not change. They will remain constant throughout the analysis which are expressed below –

$$[M_{11}] = \rho * b * h \sum_{i=1}^{nw} \sum_{j=1}^{nw} \int_0^1 \phi_i * \phi_j * d\xi \tag{4.31}$$

$$[M_{12}] = 0 \tag{4.32}$$

$$[M_{21}] = 0 \tag{4.33}$$

$$[M_{22}] = \rho * b * h \sum_{i=1+nw}^{nw+nu} \sum_{j=1+nw}^{nw+nu} \int_0^1 \alpha_{i-nw} * \alpha_{j-nw} * d\xi \tag{4.34}$$

When the displacement becomes very large then geometric non-linearity is induced in the problem. Stiffness matrix can be a function of undetermined parameters. Hence governing equations of system becomes non-linear which cannot be solved directly. To solve such equations we employed Broyden method.

4.4 LOAD VECTOR:

$$\{f_{11}\} = L \sum_{j=1}^{nw} \int_0^1 \overline{p}(\xi) \phi_i d\xi, \ \{f_{12}\} = 0, \tag{4.41}$$

CHAPTER 5.

5.1 VALIDATION:

In the present analysis non-linear system can be analyzed with the help of Broyden method. The result of present analysis is validated through comparison with previously published paper. The result of frequency response curve obtained by considering clamped-clamped condition and uniform thickness homogenous material at loading 0.134N and 2N is validated with paper of P. Ribeiro [2004] which carried out result by finite element method. The required material properties and geometric properties of beam are given below.

5.1.1 MATERIAL PROPERTIES:

Table5.1.1 (Material properties of Beam)

Material properties	Values
Young's modulus(N/m^2)	71.72 e+9
Density (kg/m^3)	2800
Poisons ratio	0.33

5.1.2 GEOMETRIC PROPERTIES OF BEAM:

Table 5.1.2 (Geometric properties of Beam)

Material dimensions	Values(m)
Width	0.02
Thickness	0.002
Length	0.406

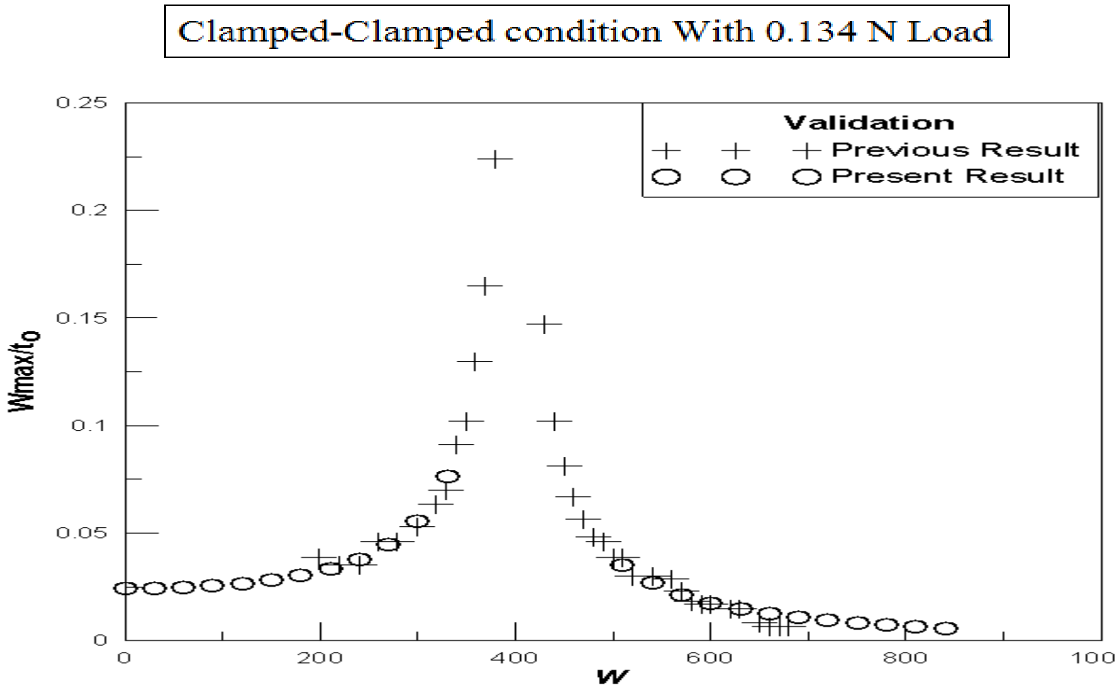


Figure 5.1 (Validation Result of 0.134 N)

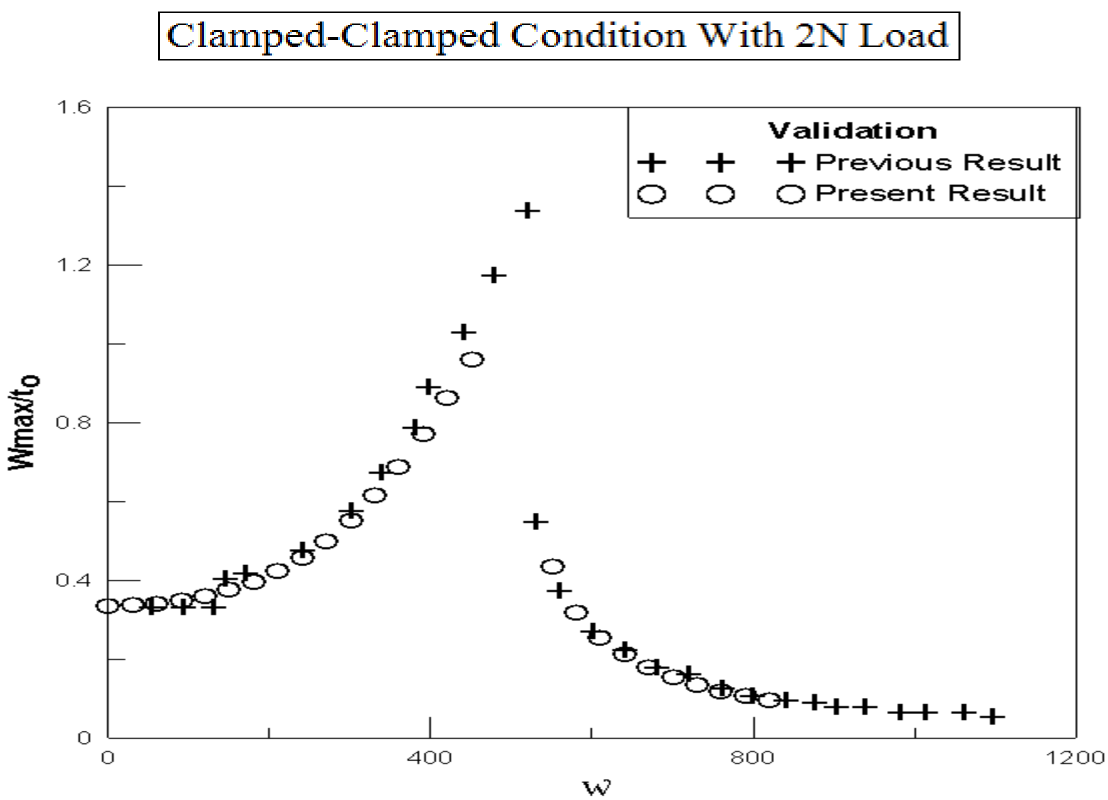


Figure 5.2(Validation Result of 2N load)

5.2 RESULTS:

In this thesis we are considering axially functionally graded material with the properties variation along longitudinal axis. By considering various boundary conditions and uniformly varying load plotted some results which represented below. The objective of the present chapter is to determine the frequency-amplitude response of non-uniform beams under transverse loading and different geometrical boundary conditions. Four boundary condition of the beam is formulated for the present analysis. First one is clamped-clamped, in which both the side of the beam are clamped or fixed. Second is clamped-simply supported. Third one is simply supported-simply supported and the last one is clamped-free (cantilever). The transverse loading for all the cases is considered to be uniformly distributed. However, the present methodology can be applied for any other transverse loading pattern, mathematically expressible as a function of the coordinates.

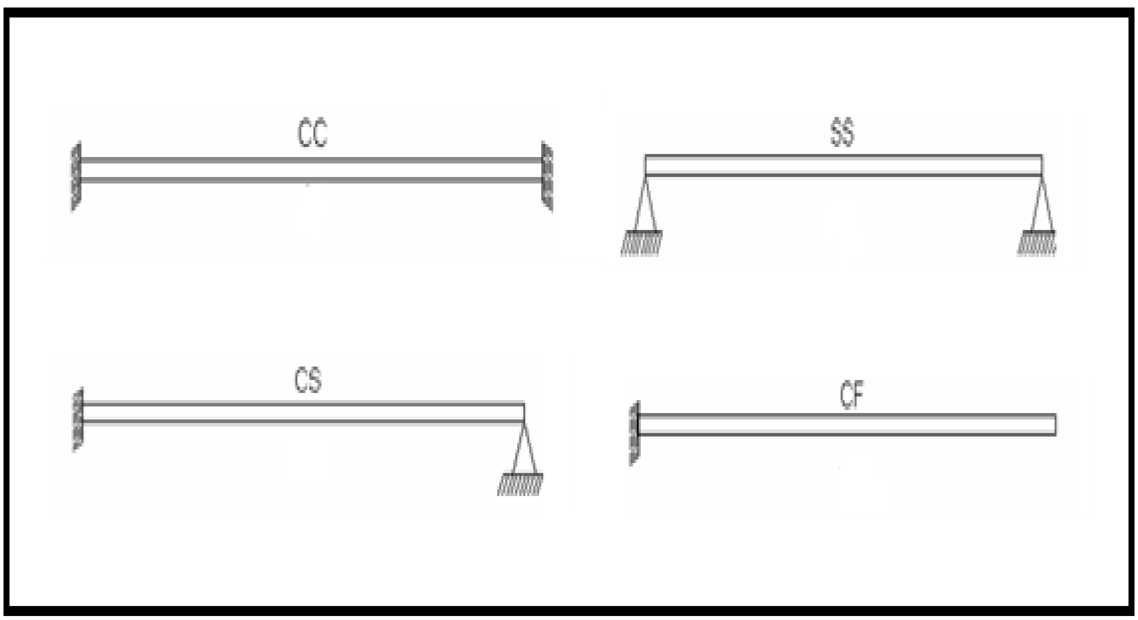


Figure 5.2.2.A (Different Boundary conditions)

Different Geometry of Beam:

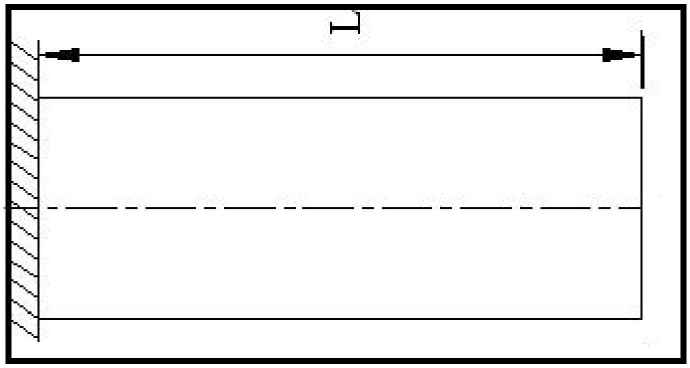


Figure 3.2.2.B (Uniform Thickness Beam)

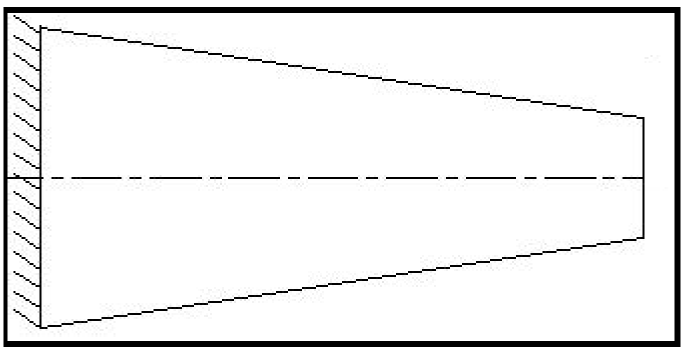


Figure 5.2.C (Linear Taper Thickness beam)

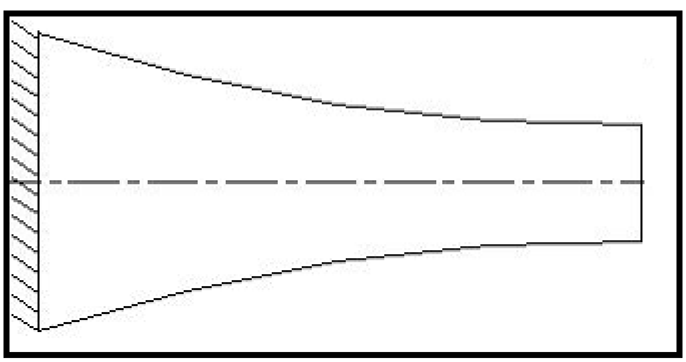


Figure 5.2.D (Exponential Thickness Beam)

In this thesis we are plotting frequency response curves for different geometry of beam and with varying material elasticity and density of beam. In the analysis we considered harmonic loading.

In the present study the important assumption considered is that system satisfies force equilibrium condition at maximum excitation amplitude. So that the sum of elastic and inertia force are equal to the externally applied load. Due to this assumption we can consider the dynamic analysis as a static case. In this analysis system response will be dependent on excitation frequency and excitation amplitude. In this case damping is considered negligible so we are assuming that system response have same frequency as external excitation. Steady state dynamic analysis is carried out by considering transverse harmonic excitation. Different frequency response curves are plotted by considering various boundary conditions.

To get the various geometric conditions following types of thickness relations are being used-

Tapereness parameter is a= 0.2

Table 5.2.A (Thickness Variation of Beam)

Uniform Thickness	thck (new) = thck (old)
Linear Tapered Thickness	thck (new) = thck (1-a* ξ)
Exponential Taper Thickness	thck (new) = thck*exp (-a* ξ)

Different types of variation of different axially functionally graded material are -

Table 5.2.B (Elasticity and density Variation of Beam)

Homogenous Elasticity	Elst (new) = Elst (old)	rho (new) = rho (old)
Linear Elasticity	Elst (new) = Elst*(1+ ξ)	rho (new) = rho*(1+ ξ + ξ^2)
Exponential Elasticity	Elst (new) = Elst*exp (ξ)	rho (new) = rho*exp (ξ)

Static analysis of beam gives the natural frequencies of beam for all mode shapes. By calculating all those natural frequencies the fundamental frequencies for various boundary condition and thickness condition are written in the tables listed below.

5.2.1 Boundary Condition

5.2.1.1 Clamped-Clamped (Fundamental frequency in rad/sec)

Table 5.2.1.1.a (Uniform Thickness CC condition)

Uniform Thickness	Homogenous Material	Linear FGM	Exponential FGM
	395.1254	391.7314	395.7343

Table 5.2.1.1.b (Linear Taper Thickness CC condition)

Linear Taper Thickness	Homogenous Material	Linear FGM	Exponential FGM
	354.5911	353.5176	356.609

Table 5.2.1.1.c (Exponential Thickness CC condition)

Exponential Thickness	Homogenous Material	Linear FGM	Exponential FGM
	354.5911	431.2346	436.2091

5.2.1.2 Clamped-Fixed (Fundamental frequency in rad/sec)

Table 5.2.1.2.a (Uniform Thickness CF condition)

Uniform Thickness	Homogenous Material	Linear FGM	Exponential FGM
	62.319	76.4748	72.4808

Table 5.2.1.2.b (Linear Taper Thickness CF condition)

Linear Taper Thickness	Homogenous Material	Linear FGM	Exponential FGM
	63.9652	78.128	74.102

Table 5.2.1.2.c (Exponential Thickness CF condition)

Exponential Thickness	Homogenous Material	Linear FGM	Exponential FGM
	60.7919	74.9079	70.9509

5.2.1.3 Clamped-Supported (Fundamental frequency in rad/sec)

Table 5.2.1.3.a (Uniform Thickness CS condition)

Uniform Thickness	Homogenous Material	Linear FGM	Exponential FGM
	272.787	283.8499	282.2727

Table 5.2.1.3.b (Linear Taper Thickness CS condition)

Linear Taper Thickness	Homogenous Material	Linear FGM	Exponential FGM
	252.2686	262.7375	261.2755

Table 5.2.3.c (Exponential Thickness CS condition)

Exponential Thickness	Homogenous Material	Linear FGM	Exponential FGM
	293.1321	304.9224	303.1916

5.2.1.4 Simply Supported (Fundamental frequency in rad/sec)

Table 5.2.1.4.a (Uniform Thickness SS condition)

Uniform Thickness	Homogenous Material	Linear FGM	Exponential FGM
	174.954	174.166	174.5244

Table 5.2.1.4.b (Linear Taper Thickness SS condition)

Linear Taper Thickness	Homogenous Material	Linear FGM	Exponential FGM
	144.1899	155.1686	155.7421

Table 5.2.1.4.c (Exponential Thickness SS condition)

Exponential Thickness	Homogenous Material	Linear FGM	Exponential FGM
	192.8407	192.9979	193.1252

5.2.2 HOMOGENOUS UNIFORM THICKNESS:

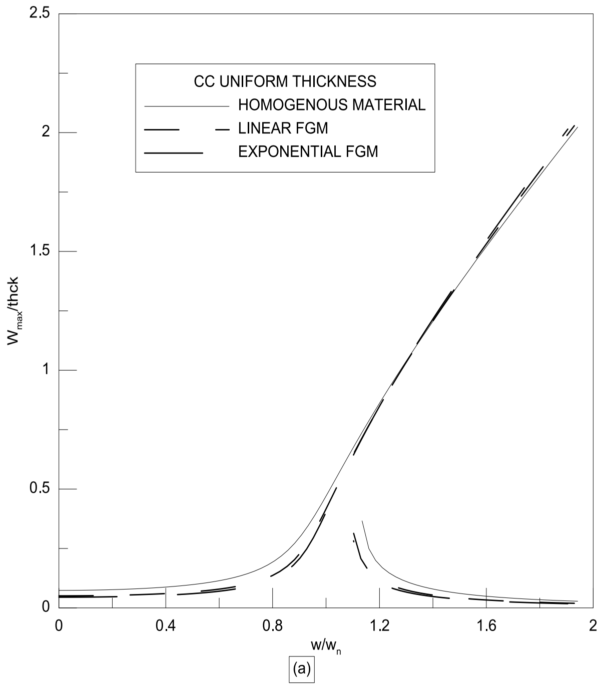


Figure 5.2.2.(a) Uniform Thickness CC condition

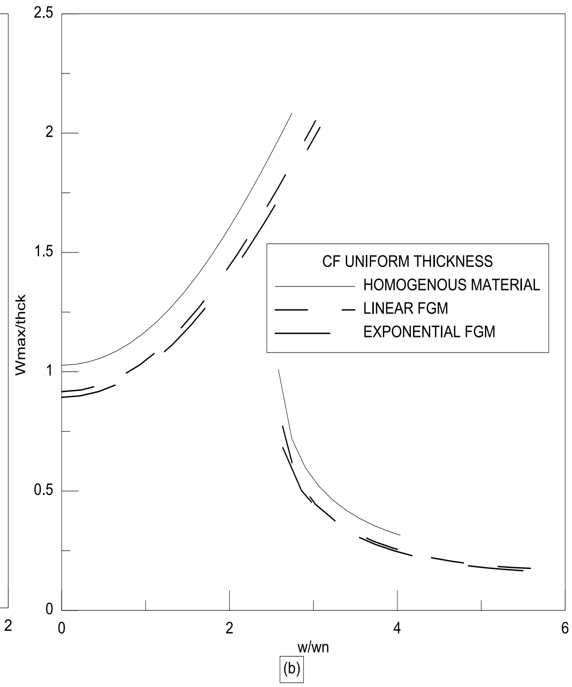


Figure 5.2.2.(b)Uniform Thickness CF condition

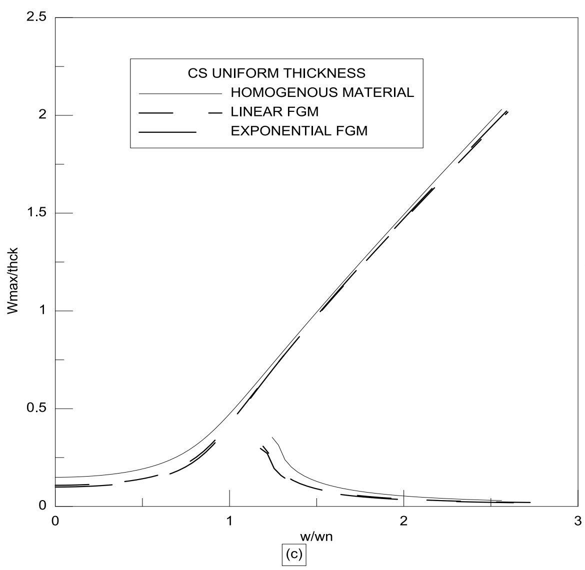


Figure 5.2.2.(c)Uniform Thickness CS condition

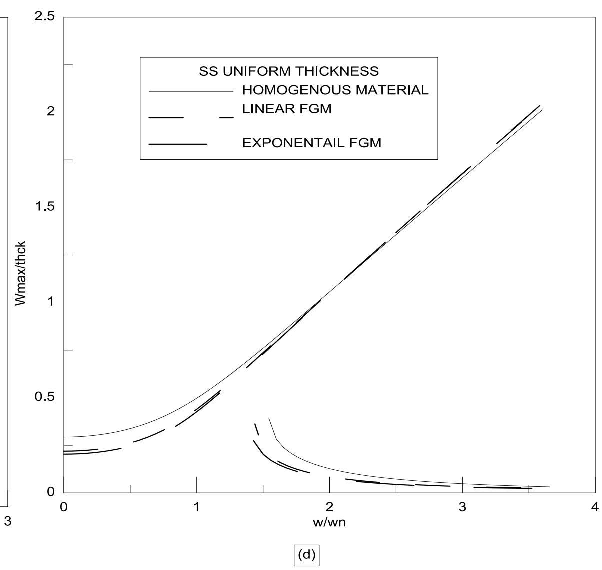


Figure 5.2.2(d) Uniform Thickness SS condition

Figure-5.2.2Non dimensional frequency response curve of uniform thickness beam under uniform distributed load (a) clamped-clamped condition (b) clamped-free condition (c) clamped-supported condition (d) simply supported condition

5.2.3 LINEAR TAPER THICKNESS:

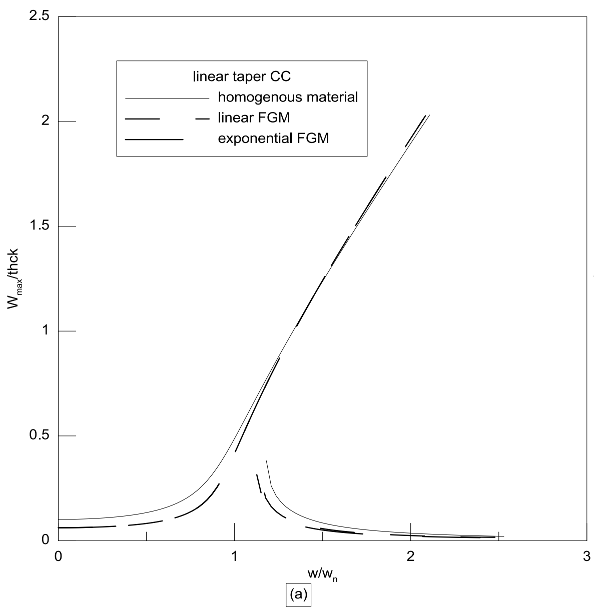


Figure 5.2.3.(a) Linear Taper Thickness CC condition

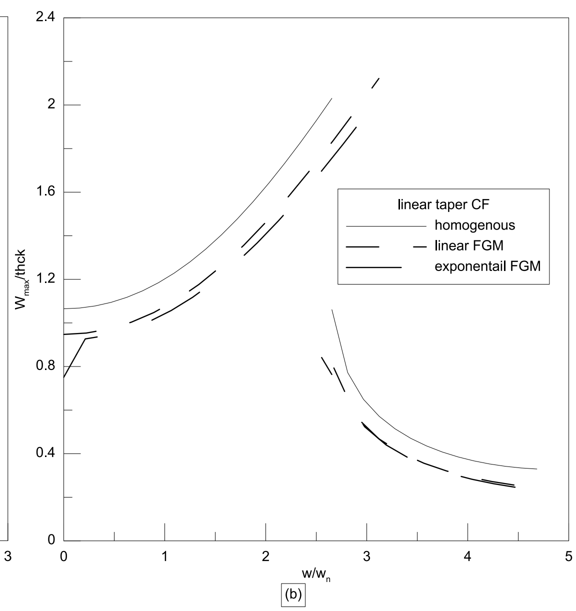


Figure 5.2.3.(b) Linear Taper Thickness CF condition

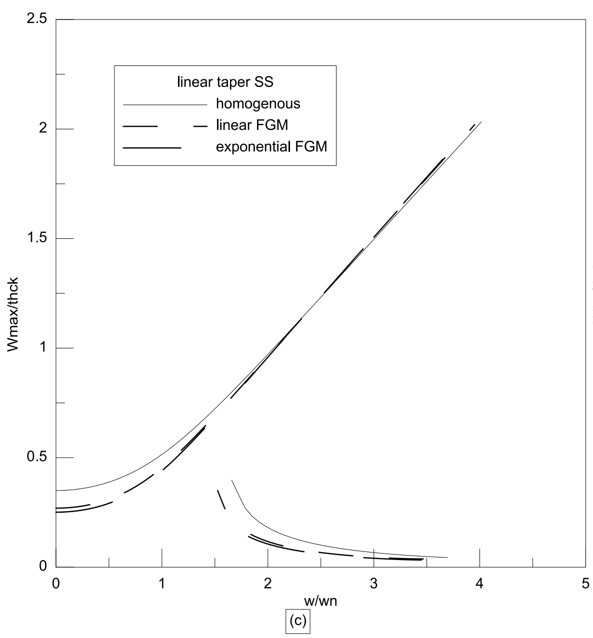


Figure 5.2.3.(c) Linear Taper Thickness SS condition

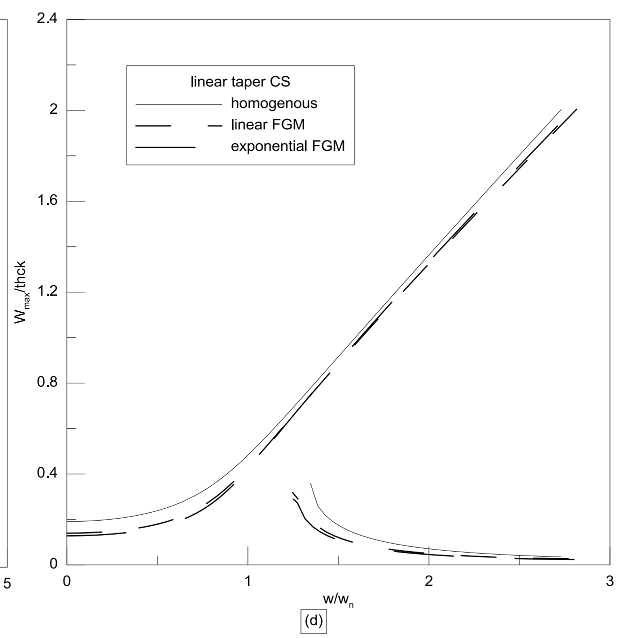
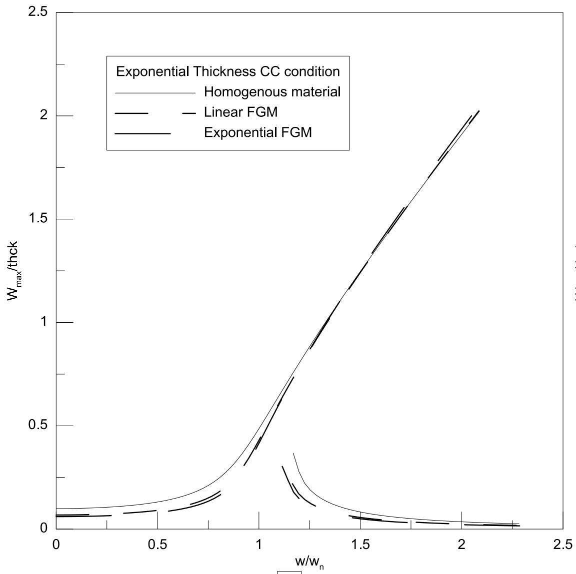


Figure 5.2.3.(d) Linear Taper Thickness CS condition

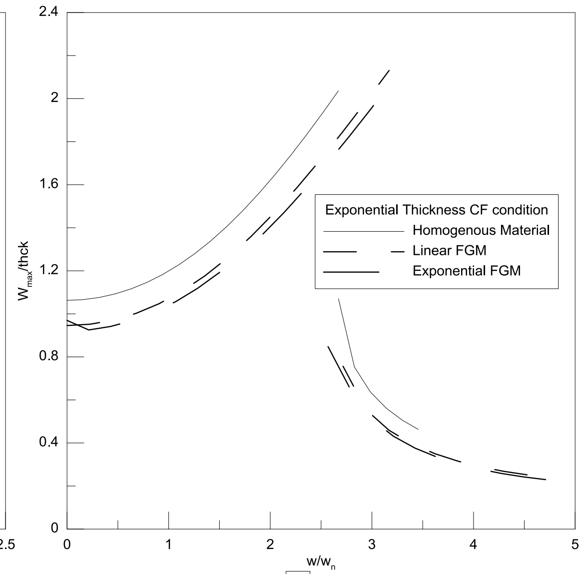
Figure-5.2.3 Non dimensional frequency response curve of linear taper thickness beam under uniform distributed load (a) clamped-clamped condition (b) clamped-free condition (c) clamped-supported condition (d) simply supported condition

5.2.4 EXOPENTIAL TAPER THICKNESS:



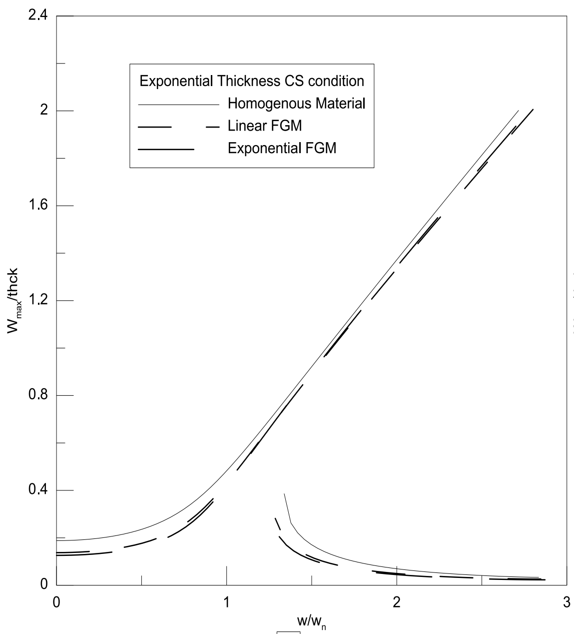
(a)

Figure 5.2.4.(a)Exponential Thickness CC condition



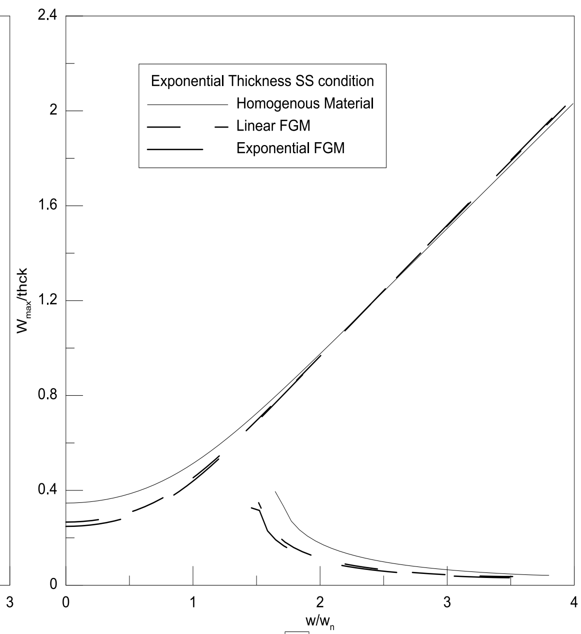
(b)

Figure 5.2.4(b) Exponential Thickness CF condition



(c)

Figure 5.2.4(c) Exponential Thickness CS condition



(d)

Figure 5.2.4(d) Exponential Thickness SS condition

Figure-5.2.4 Non dimensional frequency response curve of exponential thickness beam under uniform distributed load (a) clamped-clamped condition (b) clamped-free condition (c) clamped-supported condition (d) simply supported condition.

As per the curves plotted in the results, we can say that they exhibit nonlinearity in the sense of hardening. When we will increase frequency of free vibration then the amplitude of vibration will also increase. We plotted the results for all different types of materials whose properties varies by some law. In this graphs the obtained response curve shows the two regions. In the first region where excitation frequency is having lower value the response of system is unique. In this region amplitude of vibration increases monotonically excitation frequency increases.

After certain amount of excitation frequency we will get two different branches of amplitude responses for certain frequency range. Initial amplitude of vibration is continuously increasing as excitation frequency is increased. In second part we observed the reverse trend. As we are increasing excitation frequency the vibration amplitude goes on decreasing. If we considered the first point on vertical axis that is of zero excitation frequency we will get the static response of the beam. That point shows the static deflection of beam for corresponding excitation amplitude.

If we are considering only direct substitution technique by assuming some relaxation then increasing sweep curve of excitation frequency will capture only some part of initial part of higher vibration amplitude. But in our analysis we considered the Broyden method which covers all further lower branches of response of system completely. Broyden method is best technique such that it collects all the higher values when excitation frequency is carried out for sweep up condition. Whereas in the sweep down condition it will collects all the lower values of excitation frequency. Generally response curve is divided into three parts. From which two are considered as stable zone and they can obtain experimentally. But the third middle region can’t be obtained by experiments. It is known as unstable zone.

From the analysis we considered various boundary conditions. According to results we can conclude that as rigidity of beam increases the stiffness matrix goes on increasing which increases the natural frequency of beam. Also as rigidity increases the excitation amplitude decreases which can minimizes the vibration of beam. From all the above results we can conclude that clamped-clamped condition gives better performance under vibration condition.

CHAPTER 6.

6.1 CONCLUSION:

The free vibration study leads to the knowledge of non-linear normal modes of vibration, which provide an overall understanding of the dynamical behavior. The study of forced periodic vibrations of beams and of their stability is essential because of its fundamental nature and because it has practical applications. The methodologies here presented for free and forced vibration parametric studies can be applied to gain an overall picture of the dynamic behavior of general beams. In this work large displacement forced vibration analysis of a non-uniform beam is presented using an energy method and variational formulation. The forced vibration analysis is carried out in an indirect way, in which the dynamic system is assumed to satisfy the force equilibrium condition at maximum load value, thus reducing the problem to an equivalent static case. The solution is obtained by a multi-dimensional secant method, known as Broyden's method. The system responses indicate a hardening type nonlinearity and two stable zones of the response curve is obtained. It is seen that an increase in excitation amplitude corresponds to increase in response amplitude.

6.2 FUTURE SCOPE:

Till now presented work is carried out on the basis of nonlinear displacement of non uniform beam under both free and forced vibration case. Also we considered the analysis of axially functionally graded beam. In future we can do the analysis by considering following factors-

- In the present work we not considered the post elasticity of beam. Only elastic portion of beam is considered.
- The whole analysis is done in the absence of dampers. So further research can be carried out by using various damping systems.
- In this case study was limited up to beams only. So by presenting some results of these beams and validating we can move our further analysis for various plates and shells.

CHAPTER 7.

7.1 REFERENCES:

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